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The statistical evaluation of DNA mixtures with contributors from different ethnic groups

Received: 19 February 2001 / Accepted: 19 July 2001

Abstract The effect of a structured population on the evaluation of forensic mixed stains has been considered by the authors and others. However, in countries with multiple racial or ethnic groups, it is not uncommon that contributors to a DNA mixture are of different ethnic groups. A famous example is the OJ Simpson case in which the suspect was an African-American, the victims were Caucasian Americans and the true perpetrator(s) could be from any ethnic group(s). In this paper six common mixture cases are considered and the formulae for likelihood ratios are derived. These formulae can help forensic DNA scientists acquire a better understanding of the problem. The effect of different ethnic groups is illustrated using a case in Hong Kong.

Keywords DNA mixtures · Ethnic groups · Forensic science · Likelihood ratio · Population structure

Introduction

DNA profiling has proved to be a powerful tool for forensic human identification. Consider the following simple situation: a crime has been committed, the perpetrator has left a blood stain at the scene of the crime and a suspect has been identified. Suppose that the stain in the crime scene is typed with alleles $A_i A_j$ at a particular locus and the suspect has the same alleles. It is common to calculate the probability of a random match of DNA alleles at the crime scene and those of the suspect under the hypothesis

of innocence. Under the assumption of Hardy-Weinberg (HW) equilibrium, it can be evaluated as:

$$\begin{aligned} 2p_i p_j & \text{ for heterozygous alleles } A_i A_j, i \neq j \\ p_i^2 & \text{ for homozygous alleles } A_i A_i, \end{aligned} \quad (1.1)$$

where p_i and p_j are the proportions or frequencies for alleles A_i and A_j , respectively.

In forensic DNA analyses it is not uncommon to find DNA samples containing materials from more than one person. For example, the sample in a rape case may contain materials from the victim, her consensual sexual partner(s) and/or the perpetrator(s). The mixed stain problem is complex as commented by the second National Research Council (NRC-II 1996) report on the evaluation of forensic evidence and some forensic scientists find it difficult to assess mixed stains. The mixed stain problem was discussed by Evett et al. (1991), Weir et al. (1997) and Fukshansky and Bär (1998) and the HW equilibrium was assumed. Researchers have investigated the validity of the assumption in various ethnic or racial groups (see for example Devlin and Risch 1993 and Fung 1996). The HW law, however, is seldom exactly correct (NRC-II 1996) because dependence can have developed among alleles in the same ethnic group during the human evolutionary process. It is sensible to take this dependence structure into account in the evaluation of random match probability. A relatively simple formula for assessing the dependence was provided by Balding and Nichols (1994), which was used in recommendation 4.2 of the NRC-II (1996) report for handling the single sample problem. Recently, Curran et al. (1999) and Fung and Hu (2000a) tackled the mixed stain problem with such a dependence on the population structure.

Based on genetic and statistical justifications, Balding and Nichols (1994) showed that in general, if y copies of allele A_i have been observed among n alleles, then the probability that the next allele to be observed will be A_i is:

$$P(A_i | y A_i \text{ among } n \text{ alleles}) = \frac{y\theta + (1 - \theta)p_i}{1 + (n - 1)\theta} \quad (1.2)$$

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where θ is the coancestry coefficient, which is essentially the same as Wright's (1951) F_{st} . The latter, which is well known to population geneticists, has a genetic interpretation that measures the interpopulation variation in allele frequencies. Equation (1.2) can also be established if a state of evolutionary equilibrium has been established (Curran et al. 1999; Wright 1951). The model of Balding and Nichols (1994) is quite general and their method is endorsed by the NRC-II. It forms the basis of our derivation in this paper for taking dependence into account for mixed stain problems.

Previous reports on mixtures all regarded or assumed that contributors to the mixed stain came from one single ethnic group. In practice, however, it is not uncommon that contributors to a mixed stain belong to different ethnic or racial groups in countries with multiple ethnic groups such as the USA, UK or Singapore. A famous example is the OJ Simpson case in which the defendant was an African-American and the two victims were Caucasians and mixed stains were found in this case. The perpetrator(s) could be African-American(s), Caucasian(s), or from any other group. Extensive studies from various databases indicate that there are substantial differences in frequencies among the major ethnic groups (NRC-II 1996) and ignorance of the ethnic group information of contributors to the mixed stain may not be allowed in courtrooms and could also be misleading in assessing the weight of evidence of the mixed stains. Recently, Fukshansky and Bär (1999) and Fung and Hu (2001) studied such problems, but the HW law was assumed. Triggs et al. (2000) offered a coherent method to estimate likelihood ratios for DNA match probabilities from mixed racial populations. Buckleton et al. (1998) dealt with the situation when multiple hypotheses are postulated for mixtures.

The purpose of this paper is to evaluate the mixed stain problem with contributors of different structured ethnic groups. The likelihood ratios (LR s) for six common cases as given in Curran et al. (1999) are presented. These formulae can help forensic DNA scientists acquire a better understanding of the problem. A general formula for LR is being sought and it will be reported elsewhere.

Likelihood ratio

Suppose a crime was committed and a mixed stain was collected from the crime scene. Some crime-related persons, for example the victim and the suspect(s), were typed for their DNA characteristics. Usually, a pair of propositions, the prosecution proposition (H_p) and the defence proposition (H_d) are given to explain who the contributors to the mixed stain were. We can have propositions like, for example:

- H_p The contributors were the victim and the suspect
- H_d The contributors were the victim and one unknown person.

The likelihood ratio is often used to assign the weight of the genetic evidence in this circumstance (Weir et al. 1997; Fung and Hu 2000a, b):

$$LR = \frac{P(\text{Evidence}|H_p)}{P(\text{Evidence}|H_d)}, \quad (2.1)$$

where the evidence is all the information carried by the typed persons and that found in the mixed stain. If the likelihood ratio is L , it means that the evidence is L times more likely to have arisen under explanation H_p than under explanation H_d . Under either proposition, H_p or H_d , every typed person would be declared to have contributed to the mixed stain or not.

For the simplicity of exposition, let M denote the genetic profile of the mixed stain, which is simply a listing of the distinct alleles found in the mixed stain. We also let K denote the collection of genotypes of the known typed persons. Thus the evidence can be represented as (M, K) , and the likelihood ratio can be expressed as

$$\begin{aligned} LR &= \frac{P(M, K|H_p)}{P(M, K|H_d)} \\ &= \frac{P(K|H_p) P(M|K, H_p)}{P(K|H_d) P(M|K, H_d)} \end{aligned}$$

using the third law of probability. The terms $P(K|H_p)$ and $P(K|H_d)$ denote the probabilities of having the alleles for the typed persons. They are equal because whether H_p or H_d is true or not does not provide any information about the uncertainty of K . Hence the likelihood ratio can be simplified to:

$$LR = \frac{P(M|K, H_p)}{P(M|K, H_d)}. \quad (2.2)$$

So the evaluation of LR is reduced to the evaluation of the probability $P(M|K, H)$ for some proposition H .

Likelihood ratios for common cases

Let $A_1, A_2, \dots, A_j, \dots$ denote the alleles and let p_{ij} be the allele proportion or frequency for allele A_j in ethnic group i , $i \in G = \{a, b, \dots\}$, $j = 1, 2, \dots$, with a coancestry coefficient θ_j . We assume independence of alleles among ethnic groups. Also let K_a, K_b, \dots be the genotypes of the known contributors from groups a, b, \dots , respectively and let X_a, X_b, \dots be the genotypes of the unknown contributors from groups a, b, \dots , respectively. In the following, V is the abbreviation for the victim's genotype and S for the suspect's genotype. We are going to derive the likelihood ratios for six common mixture cases as listed in Curran et al. (1999).

Four-allele mixture, heterozygous suspect and heterozygous victim

Suppose a mixed stain M was recovered and the victim and one suspect were typed. Consider the case that $M =$

$\{A_1, A_2, A_3, A_4\}$ and the genotypes of the suspect and the victim are $S = A_1A_2$ and $V = A_3A_4$, respectively. Therefore $K = (A_1A_2, A_3A_4)$. The two alternative propositions are:

- H_p The contributors of M were the victim and the suspect
- H_d The contributors of M were the victim and one unknown person.

It can be seen that there could be three persons involved in this case, the victim, the suspect and an unknown person. The maximum number of ethnic groups to which they may belong is 3. In theory, the total number of all possible combinations about which person belongs to which ethnic group is $3^3 = 27$. Without loss of generality, we assume that the unknown contributor belongs to ethnic group a , which is termed the unknown-based viewpoint. From this viewpoint, we discuss the following situations:

- C_1 V, S and X come from group a
- C_2 V, X come from group a and S comes from group b
- C_3 S, X come from group a and V comes from group b
- C_4 X comes from group a but V and S are not from group a ,

where X is the genotype of the unknown contributor.

Under proposition H_p , it is obvious that $P(M|K, H_p) = 1$. Under H_d , from the known contributor $V = A_3A_4$, we know that the genotype of the unknown contributor for explaining the mixture $M = \{A_1, A_2, A_3, A_4\}$ must be A_1A_2 , i.e., $X_a = A_1A_2$. So $P(M|K, H_d) = P(X_a = A_1A_2|K_a, H_d)$, or simply $P(X_a = A_1A_2|K_a)$. Thus, the LR is just the reciprocal of $P(X_a = A_1A_2|K_a)$ for various K_a in $C_1 - C_4$.

If C_1 holds, $K_a = (A_1A_2, A_3A_4)$, so:

$$P(X_a = A_1A_2|K_a) = P(X_a = A_1A_2|K_a = (A_1A_2, A_3A_4)).$$

To evaluate the probability using equation (1.2), we must pass from the situation of genotypes to that of alleles. In this case, we have:

$$\begin{aligned} P(X_a = A_1A_2|K_a = (A_1A_2, A_3A_4)) \\ &= 2P(A_1, A_2|A_1, A_2, A_3, A_4) \\ &= 2 \left[\frac{\theta_a + (1 - \theta_a)p_{a1}}{1 + 3\theta_a} \right] \left[\frac{\theta_a + (1 - \theta_a)p_{a2}}{1 + 4\theta_a} \right] \\ &= 2 \frac{[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]}{(1 + 3\theta_a)(1 + 4\theta_a)}, \end{aligned}$$

where the factor 2 appears because of the two possible ways of inheritance.

When C_2 holds, $K_a = A_3A_4$, so:

$$\begin{aligned} P(X_a = A_1A_2|K_a) &= P(X_a = A_1A_2|K_a = A_3A_4) \\ &= 2 \frac{(1 - \theta_a)p_{a1}(1 - \theta_a)p_{a2}}{(1 + \theta_a)(1 + 2\theta_a)} \end{aligned}$$

If C_3 holds, $K_a = A_1A_2$, so:

$$\begin{aligned} P(X_a = A_1A_2|K_a) &= P(X_a = A_1A_2|K_a = A_1A_2) \\ &= 2 \frac{[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]}{(1 + \theta_a)(1 + 2\theta_a)}. \end{aligned}$$

When situation C_4 holds, we have $K_a = \phi$, then:

$$\begin{aligned} P(X_a = A_1A_2|K_a) &= P(X_a = A_1A_2) \\ &= 2 \left[\frac{(1 - \theta_a)p_{a1}}{1 + (0 - 1)\theta_a} \right] \left[\frac{(1 - \theta_a)p_{a2}}{1 + (1 - 1)\theta_a} \right] \\ &= 2(1 - \theta_a)p_{a1}p_{a2}. \end{aligned}$$

The LR s of these cases are presented in Table 1. It should be noted that there are just four distinct formulae by which the ethnicity of the unknown person is fixed, but each formula represents an equivalence class of possible formulae.

Three-allele mixture, homozygous victim and heterozygous suspect

Suppose that $M = \{A_1, A_2, A_3\}$, $S = A_1A_2$ and $V = A_3A_3$. The two alternative propositions are:

- H_p The contributors were the victim and the suspect
- H_d The contributors were the victim and one unknown person.

It is clear that, under proposition H_p , $P(M|K, H_p) = 1$. Under H_d , since the mixture $M = \{A_1, A_2, A_3\}$ and the known contributor $V = A_3A_3$, $X_a = A_1A_2$. Similar to the derivations for a 4-allele mixture with a heterozygous suspect and victim, we have $P(M|K, H_d) = P(X_a = A_1A_2|K_a)$ and can obtain the likelihood ratios listed in Table 2. It is noted that the LR s in Tables 1 and 2 are the same.

Table 1 Likelihood ratio for $M = \{A_1, A_2, A_3, A_4\}$, $S = A_1A_2$ and $V = A_3A_4$ with H_p the contributors were the victim and the suspect, and H_d , the contributors were the victim and one unknown person

Ethnicity		Likelihood ratio	
X	V	S	
a	a	a	$(1 + 3\theta_a)(1 + 4\theta_a)/\{2[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]\}$
a	a	b	$(1 + \theta_a)(1 + 2\theta_a)/[2(1 - \theta_a)^2p_{a1}p_{a2}]$
a	b	a	$(1 + \theta_a)(1 + 2\theta_a)/\{2[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]\}$
a	\bar{a}	\bar{a}	$1/[2(1 - \theta_a)p_{a1}p_{a2}]$

\bar{a} means not group a

Table 2 Likelihood ratio for $M = \{A_1, A_2, A_3\}$, $S = A_1A_2$ and $V = A_3A_3$ with H_p the contributors were the victim and the suspect, and H_d , the contributors were the victim and one unknown person

Ethnicity		Likelihood ratio	
X	V	S	
a	a	a	$(1 + 3\theta_a)(1 + 4\theta_a)/\{2[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]\}$
a	a	b	$(1 + \theta_a)(1 + 2\theta_a)/[2(1 - \theta_a)^2p_{a1}p_{a2}]$
a	b	a	$(1 + \theta_a)(1 + 2\theta_a)/\{2[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]\}$
a	\bar{a}	\bar{a}	$1/[2(1 - \theta_a)p_{a1}p_{a2}]$

Three-allele mixture, heterozygous victim, and homozygous suspect

Consider that the DNA profiles of the mixed stain, suspect and victim are $M = \{A_1, A_2, A_3\}$, $S = A_1A_1$, and $V = A_2A_3$, respectively. The propositions are

- H_p The contributors were the victim and the suspect
- H_d The contributors were the victim and one unknown person.

From the unknown-based viewpoint, we have the same situations C_1, \dots, C_4 as in the case of a 4-allele mixture with a heterozygous victim and suspect.

Under proposition H_p , we have $P(M|K, H_p) = 1$. Under H_d , since the mixture $M = \{A_1, A_2, A_3\}$ and the known contributor $V = A_2A_3$, X_a must be one of the following forms: A_1A_1 , A_1A_2 , or A_1A_3 . Thus, $P(M|K, H_d) = P(X_a = A_1A_1|K_a) + P(X_a = A_1A_2|K_a) + P(X_a = A_1A_3|K_a)$.

If C_1 holds, $K_a = (A_1A_1, A_2A_3)$, so:

$$\begin{aligned} P(M|K, H_d) &= P(X_a = A_1A_1|K_a = (A_1A_1, A_2A_3)) \\ &\quad + P(X_a = A_1A_2|K_a = (A_1A_1, A_2A_3)) \\ &\quad + P(X_a = A_1A_3|K_a = (A_1A_1, A_2A_3)) \\ &= \frac{[2\theta_a + (1 - \theta_a)p_{a1}][3\theta_a + (1 - \theta_a)p_{a1}]}{(1 + 3\theta_a)(1 + 4\theta_a)} \\ &\quad + 2 \frac{[2\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]}{(1 + 3\theta_a)(1 + 4\theta_a)} \\ &\quad + 2 \frac{[2\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a3}]}{(1 + 3\theta_a)(1 + 4\theta_a)} \\ &= \frac{[2\theta_a + (1 - \theta_a)p_{a1}][7\theta_a + (1 - \theta_a)(p_{a1} + 2p_{a2} + 2p_{a3})]}{(1 + 3\theta_a)(1 + 4\theta_a)}. \end{aligned}$$

The LR is the reciprocal of $P(M|K, H_d)$. The LR s when C_2 , C_3 , C_4 hold can also be obtained in a similar way (Table 3).

Four-allele mixture, heterozygous suspect and one unknown

Consider the situation that the mixed stain did not originate from the victim. Instead, the contributors of the mixed stain were two perpetrators. Suppose one suspect was arrested and we have $M = \{A_1, A_2, A_3, A_4\}$, $S = A_1A_2$. The propositions are:

- H_p The contributors were the suspect and one unknown person

Table 3 Likelihood ratio for $M = \{A_1, A_2, A_3\}$, $S = A_1A_1$ and $V = A_2A_3$ with H_p the contributors were the victim and the suspect, and H_d , the contributors were the victim and one unknown person

Ethnicity			Likelihood ratio
X	V	S	
a	a	a	$(1 + 3\theta_a)(1 + 4\theta_a)/\{[2\theta_a + (1 - \theta_a)p_{a1}][7\theta_a + (1 - \theta_a)(p_{a1} + 2p_{a2} + 2p_{a3})]\}$
a	a	b	$(1 + \theta_a)(1 + 2\theta_a)/\{(1 - \theta_a)p_{a1}[5\theta_a + (1 - \theta_a)(p_{a1} + 2p_{a2} + 2p_{a3})]\}$
a	b	a	$(1 + \theta_a)(1 + 2\theta_a)/\{[2\theta_a + (1 - \theta_a)p_{a1}][3\theta_a + (1 - \theta_a)(p_{a1} + 2p_{a2} + 2p_{a3})]\}$
a	\bar{a}	\bar{a}	$1/\{p_{a1}[\theta_a + (1 - \theta_a)(p_{a1} + 2p_{a2} + 2p_{a3})]\}$

- H_d The contributors were two unknown persons.

This is a more complicated case and the two unknown persons with genotypes X_1 and X_2 , respectively, may come from different ethnic groups. From the unknown-based viewpoint, we discuss the following situations:

- C_1 X_1, X_2 and S come from group a
- C_2 X_1 and X_2 come from group a , S comes from group b
- C_3 X_1 comes from group a , X_2 and S come from group b
- C_4 X_1 comes from group a , X_2 comes from group b , and S comes from group c
- C_5 X_1 and S come from group a , X_2 comes from group b .

Under proposition H_p , since the known contributor $S = A_1A_2$, $X_a = A_3A_4$. Thus $P(M|K, H_p) = P(X_a = A_3A_4|K_a)$. Under H_d , since the mixture $M = \{A_1, A_2, A_3, A_4\}$ and there is no known contributor, the genotypes of the two unknown persons can have six possible combinations: (A_1A_2, A_3A_4) , (A_1A_3, A_2A_4) , (A_1A_4, A_2A_3) , (A_2A_3, A_1A_4) , (A_2A_4, A_1A_3) and (A_3A_4, A_1A_2) . Thus we have:

$$\begin{aligned} P(M|K, H_d) &= P(X_a = (A_1A_2, A_3A_4)|K_a) \\ &\quad + P(X_a = (A_1A_3, A_2A_4)|K_a) \\ &\quad + P(X_a = (A_1A_4, A_2A_3)|K_a) \\ &\quad + P(X_a = (A_2A_3, A_1A_4)|K_a) \\ &\quad + P(X_a = (A_2A_4, A_1A_3)|K_a) \\ &\quad + P(X_a = (A_3A_4, A_1A_2)|K_a), \end{aligned}$$

if the two unknown persons come from the same ethnic group a , or:

$$\begin{aligned} P(M|K, H_d) &= P(X_a = A_1A_2|K_a)P(X_b = A_3A_4|K_b) \\ &\quad + P(X_a = A_1A_3|K_a)P(X_b = A_2A_4|K_b) \\ &\quad + P(X_a = A_1A_4|K_a)P(X_b = A_2A_3|K_b) \\ &\quad + P(X_a = A_2A_3|K_a)P(X_b = A_1A_4|K_b) \\ &\quad + P(X_a = A_2A_4|K_a)P(X_b = A_1A_3|K_b) \\ &\quad + P(X_a = A_3A_4|K_a)P(X_b = A_1A_2|K_b), \end{aligned}$$

if the two unknown persons come from different ethnic groups a and b , since the genotypes between ethnic groups are statistically independent.

If C_1 holds, $K_a = A_1A_2$, so:

$$\begin{aligned} P(M|K, H_p) &= P(X_a = A_3A_4|K_a = A_1A_2) \\ &= 2 \frac{(1 - \theta_a)p_{a3}(1 - \theta_a)p_{a4}}{(1 + \theta_a)(1 + 2\theta_a)}, \end{aligned}$$

and

$$P(M|K, H_p) = 24 \frac{[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]}{(1 + \theta_a)(1 + 2\theta_a)(1 + 3\theta_a)(1 + 4\theta_a)}.$$

Thus, the likelihood ratio obtained is:

$$LR = \frac{(1 + 3\theta_a)(1 + 4\theta_a)}{12[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]}.$$

The LR , when C_2 holds, can be derived in a similar way; see Table 4.

If C_3 holds, $K_a = \phi$, $K_b = A_1A_2$, thus:

$$P(M|K, H_p) = P(X_a = A_3A_4) = 2(1 - \theta_a)p_{a3}p_{a4}.$$

The denominator of the LR is more complicated. It is evaluated as:

$$\begin{aligned} P(M|K, H_d) &= P(X_a = A_1A_2)P(X_b = A_3A_4|K_b = A_1A_2) \\ &+ P(X_a = A_1A_3)P(X_b = A_2A_4|K_b = A_1A_2) \\ &+ P(X_a = A_1A_4)P(X_b = A_2A_3|K_b = A_1A_2) \\ &+ P(X_a = A_2A_3)P(X_b = A_1A_4|K_b = A_1A_2) \\ &+ P(X_a = A_2A_4)P(X_b = A_1A_3|K_b = A_1A_2) \\ &+ P(X_a = A_3A_4)P(X_b = A_1A_2|K_b = A_1A_2) \end{aligned}$$

which can be expressed as $4(1 - \theta_a)/[(1 + \theta_b)(1 + 2\theta_b)]$ multiplied by the sum of the six terms:

$$\begin{aligned} &[p_{a1}p_{a2}(1 - \theta_b)p_{b3}(1 - \theta_b)p_{b4}] \\ &+ [p_{a1}p_{a3}[\theta_b + (1 - \theta_b)p_{b2}](1 - \theta_b)p_{b4}] \\ &+ [p_{a1}p_{a4}[\theta_b + (1 - \theta_b)p_{b2}](1 - \theta_b)p_{b3}] \\ &+ [p_{a2}p_{a3}[\theta_b + (1 - \theta_b)p_{b1}](1 - \theta_b)p_{b4}] \\ &+ [p_{a2}p_{a4}[\theta_b + (1 - \theta_b)p_{b1}](1 - \theta_b)p_{b3}] \\ &+ [p_{a3}p_{a4}[\theta_b + (1 - \theta_b)p_{b1}][\theta_b + (1 - \theta_b)p_{b2}]]. \end{aligned}$$

The LR s when C_4 and C_5 hold can be obtained similarly (Table 4).

Table 4 Likelihood ratio for $M = \{A_1, A_2, A_3, A_4\}$, $S = A_1A_2$ with H_p the contributors were the suspect and one unknown person, and H_d , the contributors were two unknown persons

Ethnicity			Likelihood ratio
X_1	X_2	S	
a	a	a	$(1 + 3\theta_a)(1 + 4\theta_a)/\{12[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]\}$
a	a	b	$(1 + \theta_a)(1 + 2\theta_a)/\{12p_{a1}p_{a2}(1 - \theta_a)^2\}$
a	b	a	$(1 + \theta_b)(1 + 2\theta_b)p_{a3}p_{a4}/\{2\{p_{a1}p_{a2}(1 - \theta_b)^2p_{a3}p_{a4} + p_{a1}p_{a3}[\theta_b + (1 - \theta_b)p_{b2}](1 - \theta_b)p_{b4} + p_{a1}p_{a4}[\theta_b + (1 - \theta_b)p_{b2}](1 - \theta_b)p_{b3} + p_{a2}p_{a3}[\theta_b + (1 - \theta_b)p_{b1}](1 - \theta_b)p_{b4} + p_{a2}p_{a4}[\theta_b + (1 - \theta_b)p_{b1}](1 - \theta_b)p_{b3} + p_{a3}p_{a4}[\theta_b + (1 - \theta_b)p_{b1}][\theta_b + (1 - \theta_b)p_{b2}]\}\}$
a	b	c	$p_{a3}p_{a4}/\{2(1 - \theta_b)(p_{a1}p_{a2}p_{b3}p_{b4} + p_{a1}p_{a3}p_{b2}p_{b4} + p_{a1}p_{a4}p_{b2}p_{b4} + p_{a2}p_{a3}p_{b1}p_{b4} + p_{a2}p_{a4}p_{b1}p_{b3} + p_{a3}p_{a4}p_{b1}p_{b2})\}$
a	b	a	$(1 - \theta_a)^2p_{a3}p_{a4}/\{2(1 - \theta_b)([\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]p_{a3}p_{a4} + [\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a3}p_{b2}p_{b4} + [\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a4}p_{b2}p_{b3} + [\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a3}p_{b1}p_{b4} + [\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a4}p_{b1}p_{b3} + (1 - \theta_a)^2p_{a3}p_{a4}p_{b1}p_{b2})\}$

Three-allele mixture, heterozygous suspect, and one unknown

Suppose the mixed stain profile is $M = \{A_1, A_2, A_3\}$, the suspect profile is $S = A_1A_2$ and the propositions are:

- H_p The contributors were the suspect and one unknown person
- H_d The contributors were two unknown persons.

This case is the same as that for a 4-allele mixture with a heterozygote suspect and one unknown, except now the mixed stain only shows three alleles. If H_p is true, we have:

$$\begin{aligned} P(M|K, H_p) &= P(X_a = A_1A_3|K_a) + P(X_a = A_2A_3|K_a) \\ &+ P(X_a = A_3A_3|K_a). \end{aligned}$$

If H_d is true, the genotypes of the two unknown persons would have 12 possible combinations: (A_1A_1, A_2A_3) , (A_2A_3, A_1A_1) , (A_1A_2, A_1A_3) , (A_1A_3, A_1A_2) , (A_2A_2, A_1A_3) , (A_1A_3, A_2A_2) , (A_2A_1, A_2A_3) , (A_2A_3, A_2A_1) , (A_3A_3, A_1A_2) , (A_1A_2, A_3A_3) , (A_3A_1, A_3A_2) , and (A_3A_2, A_3A_1) and $P(M|K, H_p)$ is the sum of the conditional probabilities for these combinations. For brevity, the details of the derivation are omitted; see Table 5 for expressions of LR s.

Four-allele mixture, two heterozygous suspects

If two suspects were arrested and the DNA profiles are $M = \{A_1, A_2, A_3, A_4\}$, $S_1 = A_1A_2$ for suspect 1, and $S_2 = A_3A_4$ for suspect 2, the two propositions are:

- H_p The contributors were the two suspects
- H_d The contributors were two unknown persons.

From the unknown-based viewpoint, we discuss the following situations:

- C_1 X_1, X_2, S_1 and S_2 come from group a
- C_2 X_1, X_2 and S_1 come from group a , S_2 comes from group b
- C_3 X_1 and X_2 come from group a , S_1 and S_2 not from group a

Table 5 Likelihood ratio for $M = \{A_1, A_2, A_3\}$, $S = A_1 A_2$ with H_p the contributors were the suspect and one unknown person, and H_d , the contributors were two unknown persons

Ethnicity			Likelihood ratio
X_1	X_2	S	
a	a	a	$(1 + 3\theta_a)(1 + 4\theta_a)[5\theta_a + (1 - \theta_a)(2p_{a1} + 2p_{a2} + p_{a3})] / \{12[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}] \times [5\theta_a + (1 - \theta_a)(p_{a1} + p_{a2} + p_{a3})]\}$
a	a	b	$(1 + \theta_a)(1 + 2\theta_a)[\theta_a + (1 - \theta_a)(2p_{a1} + 2p_{a2} + p_{a3})] / \{12(1 - \theta_a)^2 p_{a1} p_{a2} [3\theta_a + (1 - \theta_a)(p_{a1} + p_{a2} + p_{a3})]\}$
a	b	b	$p_{a3}(1 + \theta_b)(1 + 2\theta_b)[\theta_a + (1 - \theta_a)(2p_{a1} + 2p_{a2} + p_{a3})] / \{2([\theta_b + (1 - \theta_b)p_{b1}][2\theta_b + (1 - \theta_b)p_{b1}]p_{a2}(1 - \theta_a)p_{a3} + [\theta_b + (1 - \theta_b)p_{b2}](1 - \theta_b)p_{b3}p_{a1}[\theta_a + (1 - \theta_a)p_{a1}] + 2[\theta_b + (1 - \theta_b)p_{b1}](1 - \theta_b)p_{b3}p_{a1}(1 - \theta_a)p_{a2} + 2[\theta_b + (1 - \theta_b)p_{b1}][\theta_b + (1 - \theta_b)p_{b2}]p_{a1}(1 - \theta_a)p_{a3} + [\theta_b + (1 - \theta_b)p_{b2}][2\theta_b + (1 - \theta_b)p_{b2}]p_{a1}(1 - \theta_a)p_{a3} + [\theta_b + (1 - \theta_b)p_{b1}](1 - \theta_b)p_{b3}p_{a2}[\theta_a + (1 - \theta_a)p_{a2}] + 2[\theta_b + (1 - \theta_b)p_{b2}](1 - \theta_b)p_{b3}p_{a2}(1 - \theta_a)p_{a1} + 2[\theta_b + (1 - \theta_b)p_{b2}][\theta_b + (1 - \theta_b)p_{b1}]p_{a2}(1 - \theta_a)p_{a3} + [\theta_b + (1 - \theta_b)p_{b3}](1 - \theta_b)p_{b3}p_{a1}(1 - \theta_a)p_{a2} + [\theta_b + (1 - \theta_b)p_{b1}][\theta_b + (1 - \theta_b)p_{b2}]p_{a3}[\theta_a + (1 - \theta_a)p_{a3}] + 2[\theta_b + (1 - \theta_b)p_{b2}](1 - \theta_b)p_{b3}p_{a3}(1 - \theta_a)p_{a1} + 2[\theta_b + (1 - \theta_b)p_{b1}](1 - \theta_b)p_{b3}p_{a3}(1 - \theta_a)p_{a2}]\}$
a	b	c	$p_{a3}[\theta_a + (1 - \theta_a)(2p_{a1} + 2p_{a2} + p_{a3})] / \{2(p_{a1}[\theta_a + (1 - \theta_a)p_{a1}]p_{b2}(1 - \theta_b)p_{b3} + p_{a2}(1 - \theta_a)p_{a3}p_{b1}[\theta_b + (1 - \theta_b)p_{b1}] + 2p_{a1}(1 - \theta_a)p_{a2}p_{b1}(1 - \theta_b)p_{b3} + 2p_{a1}(1 - \theta_a)p_{a3}p_{b1}(1 - \theta_b)p_{b2} + p_{a2}[\theta_a + (1 - \theta_a)p_{a2}]p_{b1}(1 - \theta_b)p_{b3} + p_{a1}(1 - \theta_a)p_{a3}p_{b2}[\theta_b + (1 - \theta_b)p_{b2}] + 2p_{a2}(1 - \theta_a)p_{a1}p_{b2}(1 - \theta_b)p_{b3} + 2p_{a2}(1 - \theta_a)p_{a3}p_{b2}(1 - \theta_b)p_{b1} + p_{a3}[\theta_a + (1 - \theta_a)p_{a3}]p_{b1}(1 - \theta_b)p_{b2} + p_{a1}(1 - \theta_a)p_{a2}p_{b3}[\theta_b + (1 - \theta_b)p_{b3}] + 2p_{a3}(1 - \theta_a)p_{a1}p_{b3}(1 - \theta_b)p_{b2} + 2p_{a3}(1 - \theta_a)p_{a2}p_{b3}(1 - \theta_b)p_{b1}]\}$
a	b	a	$p_{a3}(1 - \theta_a)[5\theta_a + (1 - \theta_a)(2p_{a1} + 2p_{a2} + p_{a3})] / \{2([\theta_a + (1 - \theta_a)p_{a1}][2\theta_a + (1 - \theta_a)p_{a1}]p_{b2}(1 - \theta_b)p_{b3} + [\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a3}p_{a1}[\theta_b + (1 - \theta_b)p_{b1}] + 2[\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a3}p_{b1}(1 - \theta_b)p_{b2} + 2[\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]p_{b1}(1 - \theta_b)p_{b3} + [\theta_a + (1 - \theta_a)p_{a2}][2\theta_a + (1 - \theta_a)p_{a2}]p_{b1}(1 - \theta_b)p_{b3} + [\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a3}p_{b2}(\theta_b + (1 - \theta_b)p_{b2}) + 2[\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a3}p_{b2}(1 - \theta_b)p_{b1} + 2[\theta_a + (1 - \theta_a)p_{a2}][\theta_a + (1 - \theta_a)p_{a1}]p_{b2}(1 - \theta_b)p_{b3} + [\theta_a + (1 - \theta_a)p_{a3}](1 - \theta_a)p_{a3}p_{b1}(1 - \theta_b)p_{b2} + [\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]p_{b3}[\theta_b + (1 - \theta_b)p_{b3}] + 2[\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a3}p_{b3}(1 - \theta_b)p_{b1} + 2[\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a3}p_{b3}(1 - \theta_b)p_{b2}]\}$

- C_4 X_1 and S_1 come from group a , X_2 and S_2 come from group b

- C_5 X_1 from group a , X_2 from group b and S_1 and S_2 not from groups a or b

- C_6 X_1 , S_1 and S_2 come from group a , X_2 comes from group b

- C_7 X_1 and S_1 from group a , X_2 from group b and S_2 from group c .

Under H_p , it is clear that $P(M|K, H_p) = 1$. Under H_d , since the mixture $M = \{A_1, A_2, A_3, A_4\}$ and there is no known contributor, the genotypes of the two unknown persons would have six possible combinations (see the 4-allele mixture with a heterozygote suspect and one unknown for a similar situation). The LR is the reciprocal of $P(M|K, H_d)$. Instead of having one person typed as in the 4-allele mixture with a heterozygote suspect and one unknown we

now have two persons typed. However, the LR s can be derived using a similar approach; see Table 6 for the LR s.

Case example

We consider a case in Hong Kong which was reported in Fung and Hu (2000b). The mixed stain at D3S1358 was found to be $M = \{14, 15, 17, 18\}$. A suspect was identified and his genotype was $S_1 = (14, 17)$. If the mixed stain did not originate from the victim, we may consider the following set of propositions (P1):

- H_p The contributors of the mixture were the suspect and an unknown

- H_d The contributors were two unknowns.

Suppose the arrested suspect was a Caucasian. The possible ethnic groups of the unknowns are taken as Caucasian (CA), Chinese (CH), or Filipino (PH) (the largest ethnic group in Hong Kong besides Chinese). The frequencies for alleles 14, 15, 17 and 18 are

Table 6 Likelihood ratio for $M = \{A_1, A_2, A_3, A_4\}$, $S_1 = A_1A_2$, and $S_2 = A_3A_4$ with H_p the contributors were the two suspects, and H_d the contributors were two unknown persons

Ethnicity				Likelihood ratio
X_1	X_2	S_1	S_2	
a	a	a	a	$(1 + 3\theta_a)(1 + 4\theta_a)(1 + 5\theta_a)(1 + 6\theta_a)/\{24[\theta_a + (1 - \theta_a)p_{a1}] \times [\theta_a + (1 - \theta_a)p_{a2}][\theta_a + (1 - \theta_a)p_{a3}][\theta_a + (1 - \theta_a)p_{a4}]\}$
a	a	a	b	$(1 + \theta_a)(1 + 2\theta_a)(1 + 3\theta_a)(1 + 4\theta_a)/\{24(1 - \theta_a)^2 p_{a3} p_{a4} \times [\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]\}$
a	a	\bar{a}	\bar{a}	$(1 + \theta_a)(1 + 2\theta_a)/\{24(1 - \theta_a)^3 p_{a1} p_{a2} p_{a3} p_{a4}\}$
a	b	a	b	$(1 + \theta_a)(1 + 2\theta_a)(1 + \theta_b)(1 + 2\theta_b)/\{4\{[\theta_a + (1 - \theta_a)p_{a1}] \times [\theta_a + (1 - \theta_a)p_{a2}][\theta_b + (1 - \theta_b)p_{b3}][\theta_b + (1 - \theta_b)p_{b4}] + [\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a3}(1 - \theta_b)p_{b2}[\theta_b + (1 - \theta_b)p_{b4}] + [\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a4}(1 - \theta_b)p_{b2}[\theta_b + (1 - \theta_b)p_{b3}] + [\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a3}(1 - \theta_b)p_{b1}[\theta_b + (1 - \theta_b)p_{b4}] + [\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a4}(1 - \theta_b)p_{b1}[\theta_b + (1 - \theta_b)p_{b3}] + (1 - \theta_a)^2 p_{a3} p_{a4}(1 - \theta_b)^2 p_{b1} p_{b2}\}\}$
a	b	$\bar{a}\bar{b}$	$\bar{a}\bar{b}$	$1/[4(1 - \theta_a)(1 - \theta_b)(p_{a1}p_{a2}p_{b3}p_{b4} + p_{a1}p_{a3}p_{b2}p_{b4} + p_{a1}p_{a4}p_{b2}p_{b3} + p_{a2}p_{a3}p_{b1}p_{b4} + p_{a2}p_{a4}p_{b1}p_{b3} + p_{a3}p_{a4}p_{b1}p_{b2})]$
a	b	a	a	$(1 + 3\theta_a)(1 + 4\theta_a)/\{4(1 - \theta_b) \times ([\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}]p_{b3}p_{b4} + [\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a3}]p_{b2}p_{b4} + [\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a4}]p_{b2}p_{b3} + [\theta_a + (1 - \theta_a)p_{a2}][\theta_a + (1 - \theta_a)p_{a3}]p_{b1}p_{b4} + [\theta_a + (1 - \theta_a)p_{a2}][\theta_a + (1 - \theta_a)p_{a4}]p_{b1}p_{b3} + [\theta_a + (1 - \theta_a)p_{a3}][\theta_a + (1 - \theta_a)p_{a4}]p_{b1}p_{b2})\}$
a	b	a	c	$(1 + \theta_a)(1 + 2\theta_a)/\{4(1 - \theta_b)((1 - \theta_a)^2 p_{a3} p_{a4} p_{b1} p_{b2} + [\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a4} p_{b1} p_{b3} + [\theta_a + (1 - \theta_a)p_{a2}](1 - \theta_a)p_{a3} p_{b1} p_{b4} + [\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a4} p_{b2} p_{b3} + [\theta_a + (1 - \theta_a)p_{a1}](1 - \theta_a)p_{a3} p_{b2} p_{b4} + [\theta_a + (1 - \theta_a)p_{a1}][\theta_a + (1 - \theta_a)p_{a2}][p_{b3} p_{b4}])\}$

$\bar{a}\bar{b}$ means not groups a or b

Table 7 Likelihood ratios with different ethnic groups of two unknowns for proposition sets (P_1 contributors were the suspect and an unknown, versus, contributors were two unknowns; and P_2 contributors were two suspects, versus, contributors were two unknowns)

Ethnicity			
X_1	X_2	P_1	P_2
CA	CA	1.95	36.6
CH	CH	12.3	341
PH	PH	13.0	285
CA	CH	3.10	62.9
CH	CA	2.39	62.9
CA	PH	2.72	56.0
PH	CA	2.66	56.0
CH	PH	9.43	262
PH	CH	12.0	262

given as CA: 0.187, 0.213, 0.223 and 0.127, CH: 0.033, 0.331, 0.239 and 0.056, PH: 0.026, 0.267, 0.286 and 0.088, respectively (Pu et al. 1999; Wong et al. 2001). The coancestry coefficients taken are the same value, 0.03, for all three ethnic groups. Based on the formulae given above, we obtained the LR s for various ethnic combinations of the unknowns with genotypes X_1 and X_2 . These results are listed in the column 3 of Table 7. There are great differences among the LR s. For example, the LR if the two unknowns were Filipinos is more than 6 times greater than if they were Caucasians. The ethnicity of the unknowns has a great effect on the LR .

If a second suspect (Caucasian) was identified and his profile was $S_2 = (15,18)$, we may be interested in the following set of propositions (P_2):

- H_p The contributors were the two suspects
- H_d The contributors were two unknowns.

As in the previous situation, we consider different ethnic groups for X_1 and X_2 . The formulae in the section on 4-allele mixture and

two heterozygote suspects are helpful in deriving the LR results which are shown in column 4 of Table 7. In this case, the LR has the highest value when both unknowns were Chinese, while the LR is the smallest if both were Caucasians. They are in a ratio of about 9:1. Again, the ethnicity of contributors has a substantial effect on the LR values.

Concluding remarks

When the contributors to a mixed sample come from more than one structured population, the likelihood ratio is used to assess the weight of the DNA evidence. This paper derives the LR s for six common mixture cases. The usefulness of the derived formulae is illustrated using a case-work example in Hong Kong. The results show that the ethnicity of contributors can have substantial effect on LR s.

This paper considers mixture problems with indistinguishable contributors. Sometimes, the major and minor contributors to the mixed sample can be inferred by considering the peak height of the alleles. If so, the number of possible contributors would be reduced and the problem would be simpler. The method we employed here can be used to derive the corresponding (simpler) LR s.

Acknowledgments The authors thank an associate editor and two referees for their helpful comments that greatly improved the presentation of the paper. The first author is partially supported by the Hong Kong RGC Competitive Earmarked Research Grant (HKU 7136/97H) and the second author is supported in part by the SEU Science Foundation (9207011101).

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